

Mathematical Modeling of Cylindrical Dielectric Resonator Antenna for Applications in High Speed Real-time Communication Systems

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Abstract

We are moving forward in an era where adaptive antenna arrays will be capable of identifying the direction of the incoming signal and steering the transmitted beam in appropriate directions. It has already been proposed that Dielectric Resonator Antennas (DRAs) can be good candidates for such applications. In this paper, we have carefully analyzed the theoretical model of a DRA and have proposed various mathematical methods for its analysis. The methods proposed herein can reduce the complexity of analysis and design of circuits involving DRAs.

Keywords— Dielectric Resonator Array (DRA), Antenna Arrays, Spatial Division Multiple Access (SDMA), Mathematical Modelling

I. INTRODUCTION

Since about 1970's, dielectric resonators have helped achieve the miniaturization of active and passive microwave components, such as oscillators and filters [1, 2] and have proved to be one of the driving forces behind advancement of communication systems such as Spatial Division Multiple Access

In a shielded environment, the resonators built with DRs can reach the unloaded Q factor of 20,000 at frequencies between 2 and 20 GHz. The principle of operation of the dielectric resonator can best be understood by studying the propagation of electromagnetic waves on a dielectric rod waveguide. The mathematical description and the experimental verification of the existence of these waves have been known for a long time. Their massive application, however, began with the introduction of optical fibers.

Consider the lowest modes of propagation on dielectric rod waveguides as shown in Figures 1-3 [3]. The first index denotes the number of full-period field variations in azimuthal direction, and the second one the number of radial variations.

When the first index is equal to zero, the electromagnetic field is circularly symmetric. In the cross sectional view, the field lines can be either concentric circles (e.g. the E field of the TE₀₁ mode), or the radial straight line (e.g. the H field of the same mode). For higher modes, the pure transverse electric or transverse magnetic fields cannot exist, so that both electric and magnetic fields must have non-vanishing longitudinal

components. Such modes are called hybrid electromagnetic (HEM), the lowest of them being HEM₁₁. The fields are properly expressed in terms of Bessel functions, and there exist closed form expressions for determining the wavelength and the propagation velocity of these waves.

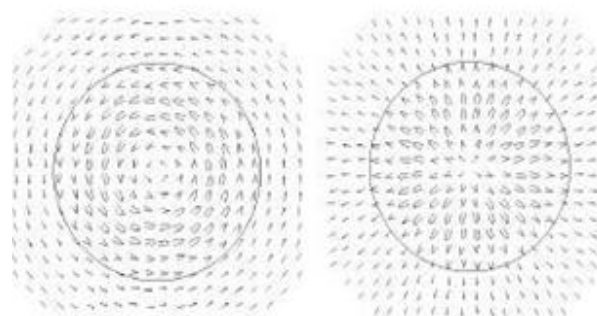


Fig. 1: Mode TE₀₁ on a dielectric rod waveguide. Left figure shows E-field while Right figure shows H-field

When only a truncated section of the dielectric rod waveguide is used, one obtains a resonant cavity in which the standing waves appear. Such a device is called dielectric resonator. When a dielectric resonator is not entirely enclosed by a conductive boundary, it can radiate, and so it becomes an antenna. DR antenna was successfully built and described in [4], while the rigorous numerical solution was published in [5]. Review treatments of DR antennas can be found in [6], [7] and [8].

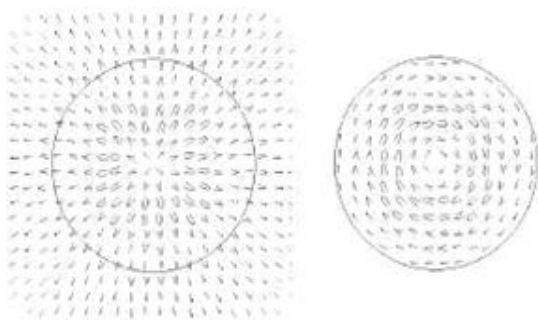


Fig. 2: Mode TM_{01} on a dielectric rod waveguide. Left figure shows E-field while Right figure shows H-field

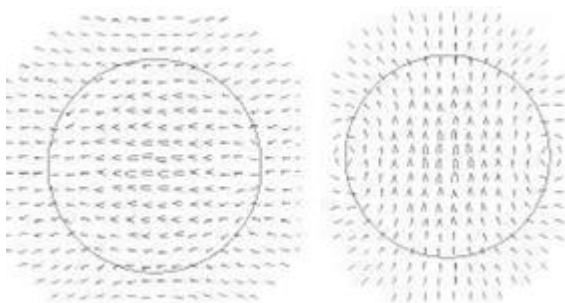


Fig. 3: Mode HEM_{11} on a dielectric rod waveguide. Left figure shows E-field while Right figure shows H-field

The numerical investigation of the DRA was started as an attempt to determine the natural frequencies of various modes of an isolated dielectric resonator, without any other scattering object in its vicinity, and without any excitation mechanism. It was found that the resonant frequencies were complex valued:

$$S_{m,n} = \sigma_{m,n} + j\omega_{m,n} \quad \dots(1)$$

Each particular solution corresponds to a resonant m, n type mode that satisfies all the boundary and continuity conditions. For rotationally symmetric resonators, subscript m denotes the number of azimuthal variations, and subscript n denotes the order of appearance of modes in the growing frequency direction.

The resonant frequency has a non-vanishing real part which signifies that such a mode would oscillate in an exponentially decaying manner, if it were initially excited by an abrupt external stimulus. The ratio of the real to the imaginary part of the natural frequency is the radiation Q factor of the mode given as:

$$Q_r = -\frac{\omega_{m,n}}{2\sigma_{m,n}} \quad \dots(2)$$

For given dimensions and dielectric constant, numerical solution can determine the resonant frequency and the radiation Q factor. Such

computed data can then be fitted to some convenient analytic expressions [9]. In this work, we proceed to find such analytic expressions which can closely simulate the actual behavior of a dielectric resonator antenna and hence can be used for real-time communication systems which are highly sensitive to various antenna parameters.

II. CYLINDRICAL DIELECTRIC RESONATOR

The general coordinate system used for the study and analysis of the cylindrical DRA is shown in Figure 4 below:

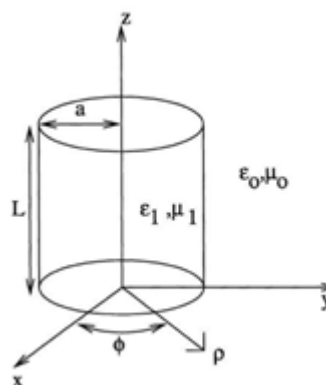


Fig. 4: General Coordinate System for Cylindrical DRA

Cylindrical DRAs are usually placed on a ground plane and is excited using a probe or aperture. It usually has an aspect ratio of about 0.5 to 4. The radiation pattern and the feeding method depend on the mode of interest. The DRA can resonate at many different modes. For the cylindrical DRA, the modes are analysed and indexed in a similar manner as the dielectric waveguide.

As is the case with dielectric waveguides, to satisfy the boundary conditions of continuity of tangential fields at the boundary of the dielectric and air, the only transverse fields that can exist are the modes with no azimuthal variation. All the other modes are hybrid modes.

A lot of conventional feeding mechanisms for DRA have been studied which are mostly derived from the theory of feeding structures for waveguides and cavities. In practice, it is typical to see DRAs being excited by a short monopole antenna or half circular current loops.

Another important aspect of the analysis of DRA is the radiation pattern. Fundamental modes of DRA radiate like magnetic or electric dipoles because of the fact that field distributions in the cavity for the low order modes support these terms. To study the radiation pattern, a nice approach is to expand the radiated fields using the multipole expansion technique which involves the decomposition of any

arbitrary radiation pattern into a sum of dipole, quadrupole and higher order multiple pole terms.

For low profile antennas operated at low order modes, the contributions of the higher order poles are weak. Generally, the smaller the radiating element is compared to the free space wavelength, the better this approximation is. The radiation pattern as a function of elevation angle for a typical circular DRA with two modes excited in quadrature phase is shown below:

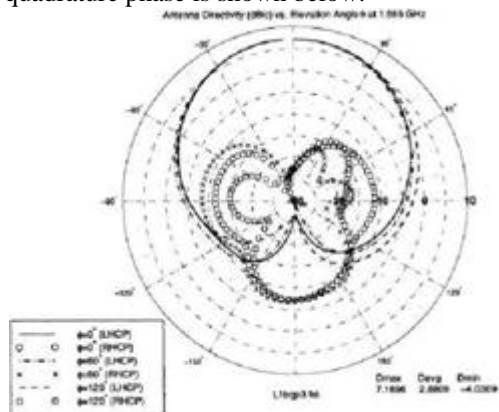


Fig. 5: Radiation Pattern as a function of elevation angle for circular DRA

The approximate field distributions inside the cylinder for the first three common modes are shown below:

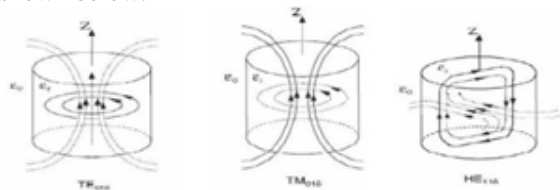


Fig. 6: Field distributions inside a cylindrical DRA

The far field radiation patterns for the TE01 modes are shown below, the second figure being the 3D equivalent of the first figure. It is clear that the electric field is doughnut-shaped.

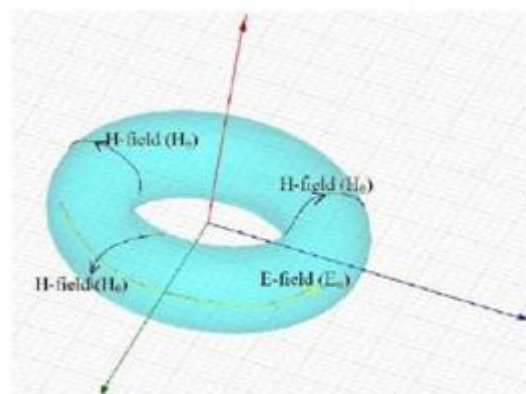
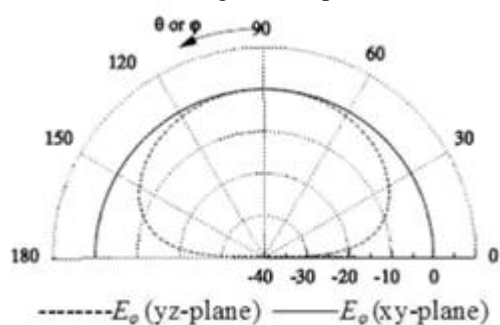


Fig. 6: Far-field radiation pattern for TE₀₁ mode

The far field radiation patterns for the TM01 modes are shown below.

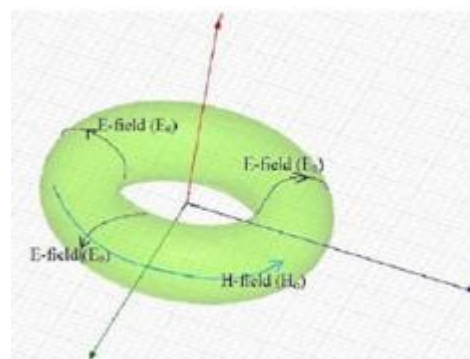
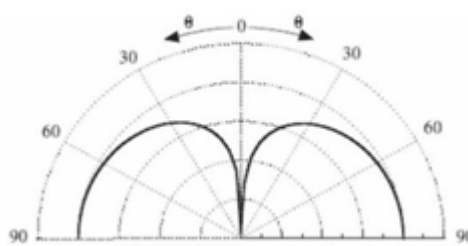


Fig. 7: Far field radiation pattern for TM₀₁ mode

Clearly, the radiation pattern of the TM₀₁ mode looks like a z-directed half-wavelength electric dipole and the TE₀₁ mode looks like a z-directed half-wavelength magnetic dipole. These kinds of insights are actually very useful towards building a mathematical model from existing mathematical frameworks.

The radiation pattern of the HE₁₁ mode resembles the radiation pattern of a half-wavelength electric dipole along the y axis and is shown below:

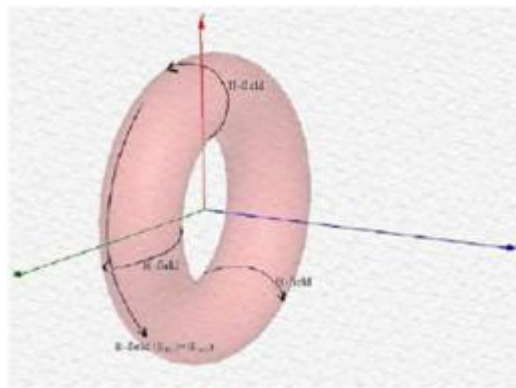
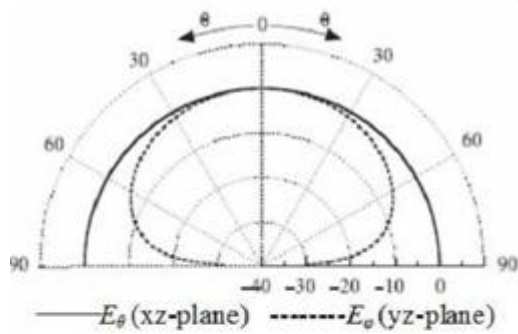


Fig. 8: Far field radiation pattern for HE₁₁ mode

III. MATHEMATICAL MODELLING

The resonant frequency for the TE₀₁ mode in a cylindrical dielectric resonator on a ground plane can be approximated as:

$$f_0 = 2.921 \cdot \frac{c\epsilon_r^{-0.465}}{2\pi a} \left[0.691 + 0.319 \frac{a}{2h} - 0.035 \cdot \left(\frac{a}{2h} \right)^2 \right] \dots (3)$$

The resonant frequency as a function of height of the cylindrical DRA at different permittivities is shown below:

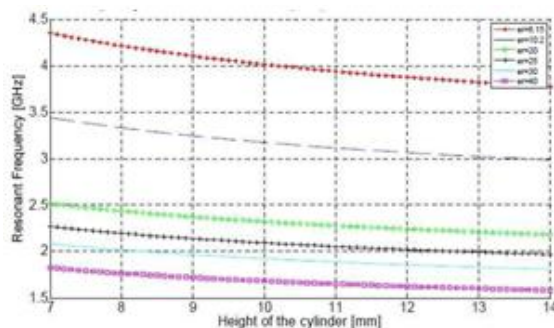


Fig. 9: Resonant frequency as a function of height at different permittivity for TE₀₁ mode

The resonant frequencies for the TM₀₁ mode can be approximated by the expression:

$$f_0 = 2.933 \cdot \frac{c\epsilon_r^{-0.468}}{2\pi a} \left[1 - \left(0.075 - 0.05 \cdot \frac{a}{2h} \right) \cdot \frac{\epsilon_r - 10}{28} \right] \left[1.048 + 0.377 \left(\frac{a}{2h} \right) - 0.071 \left(\frac{a}{2h} \right)^2 \right] \dots (4)$$

The corresponding graph of resonant frequency as a function of height of the cylindrical DRA at different permittivities for TM₀₁ mode is shown below

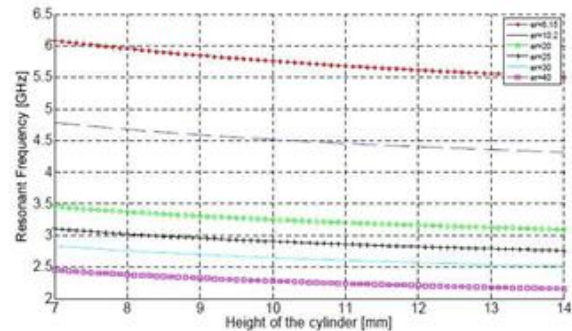


Fig. 10: Resonant frequency as a function of height at different permittivity for TM₀₁ mode

The resonant frequencies for the HE₁₁ mode can be approximated by the expression:

$$f_0 = 2.735 \cdot \frac{c\epsilon_r^{-0.436}}{2\pi a} \left[0.543 + 0.589 \frac{a}{2h} - 0.050 \cdot \left(\frac{a}{h} \right)^2 \right] \dots (5)$$

The graph of resonant frequency as a function of height of the cylindrical DRA at different permittivities for HE₁₁ mode is shown below

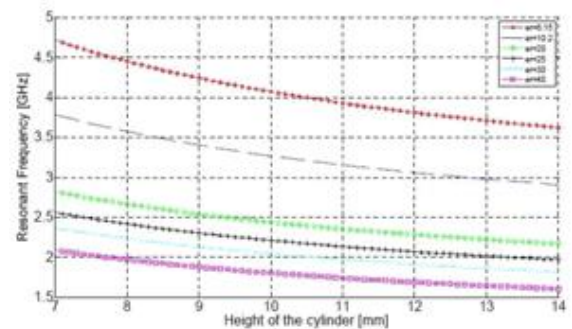


Fig. 11: Resonant frequency as a function of height at different permittivity for HE₁₁ mode

IV. CONCLUSION AND FUTURE WORK

In this work, we elucidated how cylindrical DRAs can be efficiently analysed with rigorous numerical techniques which allowed us to reproduce the graphs of the variation of their resonant frequencies as a function of cylinder height and at different permittivities. This serves as a step towards development of a rigorous mathematical algorithm for the design and analysis of cylindrical DRAs for the purpose of real-time high speed communication systems required to be operated at micro and millimetre wave ranges.

Future extension of this work will involve generalizing the numerical technique employed here to develop a software for design and analysis of

cylindrical DRAs for a number of parameter changes and varied applications.

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